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THE DYNAMIC PROBLEM OF THERMOELASTICITY FOR A HALF-SPACE WITH DISTRIBUTED HEAT SOURCES IN THE CASE OF AXIAL SYMMETRY†

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The asymptotic form as $t \to 0$ of the solution of the axisymmetric dynamic problem of thermoelasticity for a half-space heated by heat sources is obtained. The error of the asymptotic representation of the solution is estimated.

Under certain conditions, when a concentrated energy flux (a laser beam, an electron beam, etc.) acts on a solid, temperature stresses may occur in the solid, leading to its brittle fracture [1–5]. In many cases thermoelastic stresses occur due to the fact that the energy absorbed in the material gives rise to internal heat sources. Determining the stresses in the irradiated solid is the main stage in investigating the process by which fracture occurs. When using structural representations it is assumed that the fracture is due to defects (cracks), the development of which is determined by macroscopic stresses in the microdefect material.

The fracture of materials with weak absorption by concentrated energy fluxes is not determined solely by thermoelastic stresses, but occurs on the background of these for short action times [6]. Materials with intense absorption also fracture, as a rule, after a short time [1]. Hence, it is of interest to obtain a simple approximate solution of the problem of thermoelasticity for short heating times while monitoring the error of this solution.

The simplest and most natural model of an irradiated body is an elastic half-space. If the characteristic fracture time $t \gg L/c$, where L is the characteristic dimension of the region included in the fracture, and c is the velocity of propagation of elastic perturbations, then, to determine the stresses in this region one can use the quasistatic solutions for a half-space [1, 4, 7–9]. If $t \leq L/c$, the quasistatic solution is obviously insufficient and we must consider the solution of the dynamic problem.

The intensity of the concentrated energy flux in many cases falls off exponentially with depth (Bouguer's law [3]). Hence, the case of an exponential falloff with depth of the density of internal heat sources is of the greatest interest. The corresponding one-dimensional dynamic problems of thermoelasticity were considered in [10–13].

In fact, however, the intensity over the cross-section of the flow usually has a certain dome-shaped distribution. This can be approximated quite naturally by some function which falls off fairly rapidly at infinity, and which also takes into account the more complex cases of the intensity distribution characterized by the presence of several "domes" [14]. The characteristic dimension of the heating spot in fact determines the transverse dimensions of the fractured region. Hence, as far as applications are concerned, it is of the greatest interest to determine the dynamic axisymmetric state of stress in the body for short heating times.

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In [15] the axisymmetric dynamic problem for a half-space was considered with a boundary condition of the second kind and finite velocity of propagation of the heat. A relatively simple asymptotic form was obtained in the case of a point source as $t \to \infty$ for the displacements in a Rayleigh wave and as $\rho \to \infty$ (where ρ is the distance from the point of application of the thermal action) for the jumps in displacements on the elastic wave fronts. It is difficult to investigate the stress state in the neighbourhood of the point where the thermal action is applied using the results obtained in that paper.

The dynamic temperature stresses in an irradiated body are investigated below using a model of an elastic half-space in which axisymmetric distributed heat sources, which fall off with depth exponentially, act on the body. The asymptotic form as $t \to 0$ is separated from the exact solution. This paper is a continuation of the investigations carried out in [8, 9], where similar problems of thermoelasticity were considered in a quasi-static formulation, and also of the investigation carried out in [16], where a similar dynamic elastic non-temperature problem was considered.

1. In cylindrical coordinates r, φ, z we will consider an elastic half-space $z \geqslant 0$, which, up to the instant of time $t=0$, is at rest at a temperature $T=0$. From the instant of time $t=0$ distributed heat sources act in the half-space. The density of the heat sources is

$$q = q_0 f(r) e^{-\gamma z} \tag{1.1}$$

where the function $f(r)$ admits of a Hankel transformation. On the boundary of the half-space $z=0$ heat exchange occurs in accordance with Newton's law with a mean zero temperature. It is required to obtain the stresses in the half-space taking the dynamic components into account.

We will change to dimensionless quantities, assuming

$$\begin{gathered} T' = \frac{Tkc_1^2}{q_0 a^2}, \quad r' = \frac{rc_1}{a}, \quad z' = \frac{zc_1}{a}, \quad \delta' = \frac{\delta c_1}{a} \\ t' = \frac{tc_1^2}{a}, \quad h' = \frac{ha}{c_1}, \quad \alpha' = \frac{\alpha q_0 a^2}{kc_1^2}, \quad \gamma' = \frac{\gamma a}{c_1} \end{gathered} \tag{1.2}$$

where k is the thermal conductivity, a is the thermal diffusivity, h is the relative heat-exchange coefficient, α is the coefficient of linear expansion, δ is the characteristic dimension of the distribution $f(r)$, and c_1 is the velocity of longitudinal elastic waves. The primes on the dimensionless quantities will henceforth be omitted.

For the boundary-value problem of heat conduction

$$\begin{gathered} \frac{\partial T}{\partial t} = \Delta T + f(r)e^{-\gamma z}, \quad T|_{t=0} = 0, \quad \left.\frac{\partial T}{\partial z}\right|_{z=0} = hT|_{z=0} \\ |T(r,z,t)| < \infty, \quad \left(\Delta = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2}\right) \end{gathered} \tag{1.3}$$

and the transform of the temperature has the form [9]

$$\begin{gathered} T^*(r,z,s) = \int_0^\infty \lambda f^H(\lambda) J_0(\lambda r) F(\lambda,z,s)d\lambda, \quad F(\lambda,z,s) = \frac{e^{-\gamma z}}{s(\omega^2-\gamma^2)} - \frac{(\gamma+h)e^{-\omega z}}{s(\omega^2-\gamma^2)(\omega+h)} \\ T^* = L_s\{T\} = \int_0^\infty T(r,z,t)e^{-st}dt, \quad f^H(\lambda) = H_\lambda\{f(r)\} = \int_0^\infty rf(r)J_0(\lambda r)dr \\ \omega = \sqrt{s+\lambda^2}, \quad \arg\omega = 0 \text{ when } s > 0 \end{gathered} \tag{1.4}$$

where J_n is a Bessel function of the first kind.

To determine the thermoelastic potentials of the displacements it is necessary to solve boundary-value problems for the wave equations

$$\Delta\Phi - \frac{\partial^2\Phi}{\partial t^2} = m_0 T, \quad \left(\Delta - \frac{1}{r^2}\right)\Psi - \varepsilon^2 \frac{\partial^2\Psi}{\partial t^2} = 0 \tag{1.5}$$

$$\Phi|_{t=0} = \left.\frac{\partial\Phi}{\partial t}\right|_{t=0} = \Psi|_{t=0} = \left.\frac{\partial\Psi}{\partial t}\right|_{t=0} = 0, \quad |\Phi| < \infty, \quad |\Psi| < \infty$$

$$m_0 = \frac{1-\nu}{1+\nu}\alpha, \quad \varepsilon^2 = \frac{c_1^2}{c_2^2}$$

where ν is Poisson's ratio and c_2 is the velocity of transverse elastic waves.

The solutions of (1.5) can be found by means of a Laplace transformation with respect to t and Hankel transformations of the zeroth and first order with respect to r. We then obtain for the transforms of the potentials

$$\Phi^*(r,z,s) = \int_0^\infty \lambda C(\lambda,s) J_0(\lambda r) e^{-R_1 z} d\lambda -$$

$$-m_0 \int_0^\infty \lambda f^H(\lambda) J_0(\lambda r) \frac{1}{s(\omega^2-\gamma^2)} \left[\frac{e^{-\gamma z}}{R_1^2-\gamma^2} - \frac{(\gamma+h)e^{-\omega z}}{(R_1^2-\omega^2)(\omega+h)}\right] d\lambda \tag{1.6}$$

$$\Psi^*(r,z,s) = \int_0^\infty \lambda D(\lambda,s) J_1(\lambda r) e^{-R_2 z} d\lambda$$

$$R_1 = \sqrt{s^2+\lambda^2}, \quad R_2 = \sqrt{\varepsilon^2 s^2+\lambda^2}, \quad \arg R_1 = \arg R_2 = 0 \text{ when } s>0$$

By determining the transforms of the stresses corresponding to Φ^* and Ψ^* we can then obtain the unknown functions $C(\lambda, s)$ and $D(\lambda, s)$ from the boundary conditions

$$\sigma_{zz}^*\big|_{z=0} = \sigma_{rz}^*\big|_{z=0} = 0 \tag{1.7}$$

We finally obtain for the transforms of the required stresses

$$\frac{\sigma_{jj}^*}{2m_0 G} = -T^* + \frac{\nu}{1-2\nu}\int_0^\infty \lambda f^H(\lambda) J_0(\lambda r) s^2 \{Q(\lambda,z,s) + \lambda^2 \xi_3 e_1\} d\lambda +$$

$$+\int_0^\infty \lambda^3 f^H(\lambda) u_j(\lambda r) \left\{Q(\lambda,z,s) + \xi_3\left(\lambda^2 e_{12} - \frac{\varepsilon^2 s^2}{2} e_2\right)\right\} d\lambda, \quad j = r, \varphi$$

$$\frac{\sigma_{zz}^*}{2m_0 G} = \int_0^\infty \lambda f^H(\lambda) J_0(\lambda r) R^2 \{Q(\lambda,z,s) + \lambda^2 \xi_3 e_{12}\} d\lambda \tag{1.8}$$

$$\frac{\sigma_{rz}^*}{2m_0 G} = \int_0^\infty \lambda^2 f^H(\lambda) J_1(\lambda r) \left\{\gamma\xi_1 e_{R\gamma} - \omega\xi_2 e_{R\omega} + \left[\frac{P(\lambda,s)}{R_2} + \lambda^2 R_1\right]\xi_3 e_{12}\right\} d\lambda$$

$$u_r(\lambda r) = J_1(\lambda r)/(\lambda r) - J_0(\lambda r), \quad u_\varphi(\lambda r) = -J_1(\lambda r)/(\lambda r)$$

$$R^2 = \varepsilon^2 s^2/2 + \lambda^2, \quad e_1 = e^{-R_1 z}, \quad e_2 = e^{-R_2 z}, \quad e_{12} = e_1 - e_2, \quad e_{R\gamma} = e^{-R_1 z} - e^{-\gamma z}$$

$$e_{R\omega} = e^{-R_1 z} - e^{-\omega z}, \quad \xi_1 = \frac{1}{s(\omega^2-\gamma^2)(R_1^2-\gamma^2)}, \quad \xi_2 = \frac{\gamma+h}{s(\omega^2-\gamma^2)(R_1^2-\omega^2)(\omega+h)}$$

$$\xi_3 = \frac{R_2}{P(\lambda,s)}[(R_1-\gamma)\xi_1 - (R_1-\omega)\xi_2], \quad P(\lambda,s) = R^4 - \lambda^2 R_1 R_2$$

$$Q(\lambda,z,s)=\xi_1 e_{R\gamma}-\xi_2 e_{R\omega}$$

where G is the shear modulus.

The originals corresponding to expressions (1.8) can be formally written using the inversion theorem. It is difficult to use the solution obtained in this way for practical calculations.

2. When obtaining the asymptotic form as $t\to 0$ of the exact solution we will confine ourselves to the functions $f(r)$ which satisfy the following condition. We will assume a function $f^H(\lambda)$ that falls of exponentially as λ increases. This, in particular, ensures that the following integrals converge

$$A_{mn}(r)=\int_0^\infty \lambda^m f^H(\lambda)J_n(\lambda r)d\lambda,\quad m=1,2,\ldots;\ n=0,1 \tag{2.1}$$

Note that the often used distributions

$$f(r)=(r/\delta)^{2n}\exp[-r^2/(4\delta^2)],\quad f(r)=\delta^{2n+1}(r^2+\delta^2)^{-(2n+1)/2},\quad n=0,1,\ldots$$

satisfy this condition. The asymptotic expansion of σ_{zz} has the form

$$\frac{\sigma_{zz}}{2m_0G}\approx\sum_{n=0}^{\infty}A_{2n+1,0}(r)\varphi_n(z,t),\quad t\to 0 \tag{2.2}$$

We will illustrate the method of obtaining expansion (2.2) by using an example of separate terms. Consider the term

$$\sigma_{zz}^{*(1)}e^{zs}=\int_0^\infty \lambda f^H(\lambda)J_0(\lambda r)F_1(\lambda,z,s)d\lambda \tag{2.3}$$

$$F_1(\lambda,z,s)=\frac{R^2e^{-R_1z+zs}}{s(\omega^2-\gamma^2)(R_1^2-\gamma^2)}$$

After obvious reduction we have the expression

$$F_1(\lambda,z,s)=\frac{\varepsilon^2 s}{2(s-\gamma^2)(s^2-\gamma^2)}\exp\left[-\frac{\lambda^2 z}{s(1+\sqrt{1+\lambda^2/s^2})}\right]\times$$

$$\times\left(1+\frac{2\lambda^2}{\varepsilon^2 s^2}\right)\left(1+\frac{\lambda^2}{s-\gamma^2}\right)^{-1}\left(1+\frac{\lambda^2}{s^2-\gamma^2}\right)^{-1}$$

from which it follows that the series

$$F_1(\lambda,z,s)=\sum_{n=0}^{\infty}\mu_n^*(z,s)\lambda^{2n} \tag{2.4}$$

converges for any fixed value of λ and $|s|>M$. The functions $\mu_n^*(z,s)$ are proper rational fractions in s and are represented in the neighbourhood of $s=\infty$ in the form of Laurent series. By the first expansion theorem [17] we have

$$\mu_n(z,t)=L_t^{-1}\{\mu_n^*(z,s)\}=L_t^{-1}\left\{\frac{d_n(z)}{s^{n+2}}+\ldots\right\}=\frac{d_n(z)}{(n+1)!}t^{n+1}+\ldots$$

$$\mu_{n+1}(z,t)=o(\mu_n(z,t)),\quad t\to 0,\quad n=0,1,\ldots \tag{2.5}$$

where L_t^{-1} is the inverse operator to L_s. Substituting (2.4) into (2.3) and integrating term by term with respect to λ, we obtain the formal expansion

$$\sigma_{zz}^{*(1)}e^{zs} \approx \sum_{n=0}^{\infty} A_{2n+1,0}(r)\mu_n^*(z,s)$$

Changing to the originals we obtain, using the retardation theorem

$$\sigma_{zz}^{(1)}(r,z,t) \approx \sum_{n=0}^{\infty} A_{2n+1,0}(r)\mu_n(z,t-z), \quad t\to 0, \quad t>z \tag{2.6}$$

It can be proved that (2.6) is, in fact, the asymptotic expansion of $\sigma_{zz}^{(1)}$ as $t\to 0$.

The first condition of the asymptotic expansion is satisfied by virtue of (2.5). We will estimate the order of smallness of $r_n(r, z, t)$ as $t\to 0$, which is equal to the difference between the left-hand side and the first $n+1$ terms on the right-hand side of relation (2.6). As a consequence of relations (2.1) and (2.3) we will have

$$r_n(r,z,t+z) = \int_0^{\infty} \lambda f^H(\lambda)J_0(\lambda r)L_t^{-1}\{q_n^*(\lambda,z,s)\}d\lambda$$

$$q_n^*(\lambda,z,s) = F_1(\lambda,z,s) - \sum_{k=0}^{n} \mu_k^*(z,s)\lambda^{2k}$$

where the possibility of changing to the original under the integral sign is ensured by the uniform convergence of the corresponding integrals established on the assumption of an exponential decrease in $f^H(\lambda)$.

From relations (2.4) and (2.5) we have, when $|s|>M$ the expansion

$$q_n^*(\lambda,z,s) = b_{n+1}(\lambda,z)/s^{n+3} + \dots$$

using which we obtain

$$r_n(r,z,t) = o(\mu_n(z,t-z)), \quad t\to 0, \quad t>z \tag{2.7}$$

which indicates that the second condition of the asymptotic expansion is satisfied.

The asymptotic expansions of the originals of the terms containing the factors $e^{-R_1 z}$, $e^{-R_2 z}$, $e^{-\gamma z}$ are obtained in the same way as for $\sigma_{zz}^{(1)}$, since the roots of the equation $P(\lambda, s)=0$ are only $s=0$ and $s=\pm i\lambda\vartheta/\varepsilon$, $0<\vartheta<1$ [18]. For the transforms having the factor $(\omega+h)^{-1}$ in relations similar to (2.5), instead of the first expansion theorem its generalization for fractional powers of s is employed [19].

To obtain the asymptotic expansion of the original of the term having the factor $e^{-\omega z}$, we will use a method which is essentially a generalization of the method from [20].

The original has the form

$$\sigma_{zz}^{(2)} = (\gamma+h)[(\varepsilon^2/2)P_1(r,z,t) + P_2(r,z,t)] \tag{2.8}$$

$$P_k(r,z,t) = \int_0^{\infty} \lambda^{2k-1} f^H(\lambda)J_0(\lambda r)L_t^{-1}\left\{\frac{\zeta(z,s+\lambda^2)}{s^{2k-2}(s-1)}\right\}d\lambda, \quad k=1,2$$

$$\zeta(z,s) = e^{-z\sqrt{s}}(s-\gamma^2)^{-1}(\sqrt{s}+h)^{-1}$$

For $P_1(r, z, t)$, using the convolution and displacement theorems and then expanding $e^{-\lambda^2\tau}$ using Taylor's formula, changing the order of integration with respect to λ and τ and integrating with respect to λ term by term, we will have

$$P_1(r,z,t) = \int_0^{\infty} \lambda f^H(\lambda)J_0(\lambda r)d\lambda \int_0^{t} e^{t-\tau}e^{-\lambda^2\tau}L_\tau^{-1}\{\zeta(z,s)\}d\tau =$$

$$= \sum_{k=0}^{n} A_{2k+1,0}(r)\rho_k(z,t)+g_n(r,z,t)$$

$$\rho_k(z,t)=\frac{(-1)^k}{k!}e^t\int_0^t\tau^k L_\tau^{-1}\{\zeta(z,s+1)\}d\tau=\sum_{m=0}^{k}(-1)^{k+m}\frac{t^{k-m}}{(k-m)!}L_t^{-1}\left\{\frac{\zeta(z,s)}{(s-1)^{m+1}}\right\} \tag{2.9}$$

$$g_n(r,z,t)=e^t\frac{(-1)^{n+1}}{(n+1)!}\int_0^t\tau^{n+1}L_\tau^{-1}\{\zeta(z,s+1)\}d\tau\int_0^\infty\lambda^{2n+2}f^H(\lambda)e^{-\lambda^2\vartheta_1\tau}J_0(\lambda r)d\lambda$$

$$0<\vartheta_1<1$$

The last expression for $\rho_k(z, t)$ is obtained by k-fold integration by parts using the displacement theorem.

Applying the generalized theorem on the mean [21] ($L_t^{-1}\{\xi(z, s)\}\geqslant 0$ [20]) to the integral representation of $\rho_{k+1}(z, t)$ we obtain

$$\rho_{k+1}(z,t)=-\frac{\vartheta_2 t}{k+1}\rho_k(z,t),\quad 0<\vartheta_2<1$$

$$\rho_{k+1}(z,t)=o(\rho_k(z,t)),\quad t\to 0$$

$$|g_n(r,z,t)|\leqslant|\rho_{n+1}(z,t)|\int_0^\infty\lambda^{2n+2}|f^H(\lambda)|d\lambda \tag{2.10}$$

$$g_n(r,z,t)=o(\rho_n(z,t)),\quad t\to 0$$

It follows from (2.10) and (2.9) that

$$P_1(r,z,t)\sim\sum_{k=0}^{\infty}A_{2k+1,0}(r)\rho_k(z,t),\quad t\to 0 \tag{2.11}$$

The asymptotic expansion for $P_2(r, z, t)$ is obtained by replacing $A_{2k+1,0}(r)$ by $A_{2k+3,0}(r)$ in (2.11) and by double integration in the section $[0, t]$.

Adding all the asymptotic expansions of the terms which form σ_{zz} and combining terms with the same factors $A_{2n+1,0}(r)$ we obtain relation (2.2).

Similar expansions can also be obtained for the residual stresses.

Since $c_1\sim 10^3$ m/s and $a\sim 10^{-6}$ m²/s, it follows from the fifth relation of (1.2) that large values of the dimensionless time t' may correspond to physically small times t. Hence, the use of the asymptotic form as $t'\to 0$ to obtain an approximate solution requires some justification. In this connection we will consider some features of the asymptotic expansion of (2.2). We will revert for a moment denoting dimensionless quantities by primed letters. Since the function $f(r)$ is dimensionless, $f(r)=f_1(r/\delta)=f_1(r'/\delta')$.

From (2.1) and the properties of the Hankel transformation successively we obtain

$$f^H(\lambda)=\delta'^2 f_1^H(\lambda\delta'),\quad A_{mn}(r)=\frac{1}{\delta'^{m-1}}A_{mn}^{(0)}\left(\frac{r'}{\delta'}\right) \tag{2.12}$$

where $A_{mn}^{(0)}(r'/\delta')$ has the same values in dimensional and dimensionless quantities.

The transformations $\varphi_n^*(z', s)=L_s\{\varphi_n(z', t')\}$ are the coefficients of the expansion in a power series in λ^2 of the function

$$\varphi^*(\lambda^2,z',s)=R^2[Q(\lambda,z',s)+\lambda^2\xi_3 e_{12}]$$

It can be shown that the function $\varphi^*(\lambda^2, z, s)$ and all its derivatives with respect to λ^2 have only removable singular points in the half-plane $\operatorname{Re}s>0$.

The behaviour of $\varphi_n(z', t')$ as $t'\to\infty$ is determined by the expansion of $\varphi_n^*(z', s)$ in the neighbourhood of the singular point $s=0$. Hence, using Theorem 35.1 of [19], it can be proved that the following relations hold

$$\varphi_n(z',t')=t'^{2n}\psi_n(z',t') \tag{2.13}$$

where $\psi_n(z', t')$ are bounded as $t'\to\infty$. Consequently, from (2.2), (2.12) and (2.13) we obtain the expansion

$$\frac{\sigma_{zz}}{2m_0G}\approx\sum_{n=0}^{\infty}\left(\frac{t'}{\delta'}\right)^{2n}A^{(0)}_{2n+1,0}\left(\frac{r'}{\delta'}\right)\psi_n(z',t'),\quad t'\to 0 \tag{2.14}$$

Since $t'\delta'=t_1=c_1t/\delta$, while the coefficients of t_1^{2n} in the expansion (2.14) are bounded as $t'\to\infty$, we would expect the asymptotic form obtained from (2.14) by retaining a finite number of terms to be fairly accurate when $c_1t/\delta\ll 1$. Finally, the quality of the asymptotic form obtained can be established by estimating the error.

We will confine ourselves to asymptotic representations of the exact solution. Once again omitting the primes when denoting dimensionless quantities, we will have

$$T(r,z,t)=T^{(0)}+\delta_T=f(r)L_t^{-1}\left\{\frac{e^{-\gamma z}}{s(s-\gamma^2)}-\frac{(\gamma+h)e^{-z\sqrt{s}}}{s(s-\gamma^2)(\sqrt{s}+h)}\right\}+\delta_T$$

$$\frac{\sigma_{jj}}{2m_0G}=-k_jT^{(0)}+l_jf(r)L_t^{-1}\left\{\frac{s(e^{-zs}-e^{-\gamma z})}{(s-\gamma^2)(s^2-\gamma^2)}-\frac{(\gamma+h)(e^{-zs}-e^{-z\sqrt{s}})}{(s-\gamma^2)(s-1)(\sqrt{s}+h)}\right\}+\delta_{jj}$$

$$\frac{\sigma_{rz}}{2m_0G}=A_{21}(r)L_t^{-1}\left\{\frac{e^{-zs}-e^{-\varepsilon zs}}{s(s-\gamma^2)(s+\gamma)}-\frac{(\gamma+h)(e^{-zs}-e^{-\varepsilon zs})}{s\sqrt{s}(s-\gamma^2)(\sqrt{s}+1)(\sqrt{s}+h)}+\right. \tag{2.15}$$

$$\left.+\frac{\gamma(e^{-zs}-e^{-\gamma z})}{s(s-\gamma^2)(s^2-\gamma^2)}-\frac{(\gamma+h)(e^{-zs}-e^{-z\sqrt{s}})}{s\sqrt{s}(s-\gamma^2)(s-1)(\sqrt{s}+h)}\right\}+\delta_{rz}$$

$$j=r,\varphi,z,\quad k_r=k_\varphi=1,\quad k_z=0,\quad l_r=l_\varphi=\nu/(1-2\nu),\quad l_z=\varepsilon^2/2$$

Relations (2.15) are exact expressions for the stresses. We can obtain an approximate solution by dropping the errors δ_{ij}, $(i,\ j=r,\ \varphi,\ z)$, δ_T in (2.15). Note that the temperature and the normal stresses in the approximate solution are the solution of the corresponding one-dimensional problem multiplied by $f(r)$.

3. We will estimate δ_T. Using the theorem of the integration of an original and the displacement we obtain from (1.4) and (2.15)

$$|\delta_T|=\left|\int_0^{\infty}\lambda f^H(\lambda)J_0(\lambda r)d\lambda\int_0^t(e^{-\lambda^2\tau}-1)L_\tau^{-1}\{sF(0,z,s)\}d\tau\right| \tag{3.1}$$

Since $L_t^{-1}\{sF(0,\ z,\ s)\}>0$ like the derivative with respect to t of the solution of the one-dimensional boundary-value problem of heat conduction, which is obtained from (1.3) when $f(r)=1$, $\Delta=\partial^2/\partial z^2$ and

$$L_t^{-1}\left\{\frac{(\gamma+h)e^{-z\sqrt{s}}}{(s-\gamma^2)(\sqrt{s}+h)}\right\}\geqslant 0$$

and like the convolution of positive originals [20], we have

$$L_t^{-1}\left\{\frac{(\gamma+h)e^{-z\sqrt{s}}}{(s-\gamma^2)(\sqrt{s}+h)}\right\}<L_t^{-1}\left\{\frac{e^{-\gamma z}}{s-\gamma^2}\right\}=e^{\gamma^2 t-\gamma z}$$

$$L_t^{-1}\{sF(0,z,s)\}\leq e^{\gamma^2 t-\gamma z} \tag{3.2}$$

From (3.1) and (3.2) using the inequalities $|J_0(x)|\leq 1,\ 1-e^{-\lambda^2 t}\leq\lambda^2 t$ we obtain

$$|\delta_T|\leq\frac{\beta_3 t^2}{2}e^{\gamma^2 t-\gamma z},\quad \beta_n=\int_0^\infty\lambda^n|f^H(\lambda)|d\lambda,\quad n=1,2\ldots \tag{3.3}$$

As was pointed out in Section 2, large values of the dimensionless time t may correspond to physically small times. In order to obtain satisfactory estimates of the errors δ_{ij} $(i,\ j=r,\ \varphi,\ z)$ for fairly large intervals t, it was necessary to take some care when deriving power estimates of the form At^μ. It is desirable to obtain the smallest possible value of μ for which, however, the natural condition $At^\mu=o(\sigma_{ij}^{(0)}),\ t\to 0$ is satisfied ($\sigma_{ij}^{(0)}$ is the asymptotic representation of σ_{ij}). This consideration is made use of when deriving the following fundamental inequalities

$$\left|L_t^{-1}\left\{\frac{1}{R_1+\gamma}-\frac{1}{s+\gamma}\right\}\right|\leq\frac{\lambda^2 t^2}{4},\quad |L_t^{-1}\{\chi(\lambda,\gamma)\}|\leq\eta_0+z(\gamma+\lambda^2 t)\eta_1$$

$$|L_t^{-1}\{\chi(\lambda,\gamma)-\chi(0,\gamma)\}|\leq(\lambda^2t^2/2)\eta_0+(\lambda^2 zt/2)[1+\gamma(t-z)]\eta_1$$

$$\left|L_t^{-1}\left\{\frac{e^{-R_1 z}-e^{-R_2 z}}{R_1}\right\}\right|\leq\eta_1-\eta_2+\lambda^2 z\left(z+\frac{\varepsilon^2-1}{2\varepsilon}t\right)\eta_2$$

$$|L_t^{-1}\{\chi(\lambda,\omega)\}|\leq e^{t-z}\eta_0+z[(\pi t)^{-1/2}e^{-z^2/(4t)}+\lambda^2\kappa]\eta_1 \tag{3.4}$$

$$|L_t^{-1}\{\chi(\lambda,\omega)-\chi(0,\sqrt{s})\}|\leq\lambda^2 te^{t-z}\eta_0+\lambda^2 z\kappa\eta_1,\quad |L_t^{-1}\{(R_1+\omega)^{-1}\}|\leq 1+2\lambda$$

$$\left|L_t^{-1}\left\{\frac{1}{R_1+\omega}-\frac{1}{s+\sqrt{s}}\right\}\right|\leq\lambda^2\left(2t+\frac{t^2}{2}\right),\quad \left|L_t^{-1}\left\{\frac{sR_1R_2}{P(\lambda,s)}\right\}\right|\leq\beta$$

$$\left|L_t^{-1}\left\{\frac{e^{-R_1 z}-e^{-R_2 z}}{s}\right\}\right|\leq\eta_1-\eta_2+\frac{\lambda^2 z}{2}\left[(t-z)\eta_1+\frac{1}{\varepsilon}(t-\varepsilon z)\eta_2\right]$$

$$|J_0^{(2n)}(x)|\leq\frac{(2n-1)!!}{(2n)!!},\quad |J_0^{(2n-1)}(x)|\leq\frac{2(2n-2)!!}{\pi(2n-1)!!},\quad n=1,2,\ldots$$

$$\chi(x,y)=\frac{s(e^{-z\sqrt{s^2+x^2}}-e^{-yz})}{s^2+x^2-y^2}$$

$$\eta_1=\eta(t-z),\quad \eta_2=\eta(t-\varepsilon z),\quad \eta_0=1-\eta_1,\quad \kappa=1+\sqrt{t/2}$$

$$\beta=\frac{2}{\varepsilon^2-1}+\frac{\varepsilon^2-1}{2\pi\varepsilon^2}+\frac{1}{\varepsilon^2}\frac{8(\varepsilon^2-\vartheta^2)(1-\vartheta^2)}{\varepsilon^2\vartheta^6-6\varepsilon^2\vartheta^4+(12\varepsilon^2-8)\vartheta^2-4(\varepsilon^2-1)}$$

where $\eta(x)$ is the Heaviside unit function, and $\pm i\vartheta/\varepsilon$ are the non-zero roots of the equation $P(1,\ s)=0$.

The first four inequalities of (3.4) are derived using the well-known relation between the originals of the transformations $g(s)$ and $g(\sqrt{(s^2+\lambda^2)})$ [22] and fundamental theorems of the operational calculus. For the first relation of (3.4), for example, using the inequality $|J_1(x)|\leq|x|/2$, we have

$$\left|L_t^{-1}\left\{\frac{1}{R_1+\gamma}-\frac{1}{s+\gamma}\right\}\right|=\left|\lambda\int_0^t e^{-\gamma\sqrt{t^2-u^2}}J_1(\lambda u)du\right|\leq\frac{\lambda^2 t^2}{4}$$

The inversion theorem is used to derive the following five relations of (3.4). For the first of these we have, when $0 \leqslant t < z$, using the displacement theorem, the originals $L_t^{-1}\{e^{-z\sqrt{s}}\}$ and $L_t^{-1}\{e^{-z\sqrt{s}}(s-1)^{-1}\}$ from [23] and Eq. (3.546.2) from [24]

$$
\begin{aligned}
&L_t^{-1}\left\{\frac{se^{-\omega z}}{R_1^2-\omega^2}\right\} = \int_0^t e^{t-\tau}e^{-\lambda^2\tau}L_\tau^{-1}\{e^{-z\sqrt{s}}\}d\tau \leqslant L_t^{-1}\left\{\frac{e^{-z\sqrt{s}}}{s-1}\right\} = \\
&= \frac{1}{\sqrt{\pi}}e_z\int_0^\infty [\exp(-x^2+2x\sqrt{t}-zx/\sqrt{t})+\exp(-x^2-2x\sqrt{t}-zx/\sqrt{t})]dx \leqslant \\
&\leqslant \frac{2}{\sqrt{\pi}}e_z\int_0^\infty e^{-x^2}\operatorname{ch}[x(2\sqrt{t}-z/\sqrt{t})]dx = e^{t-z}, \quad e_z = e^{-z^2/(4t)}
\end{aligned}
\tag{3.5}
$$

Suppose now that $t \geqslant z$. The transformation in the fifth relation of (3.4) is a regular function of s in a plane with cuts $(-i\infty, -i\lambda]$, $[i\lambda, i\infty)$, $(-\infty, -\lambda^2]$. Using the inversion theorem and Jordan's lemma we can reduce the integral over the straight line $\operatorname{Re} s = \sigma$ to a set of integrals over the edges of the cuts. Taking into account the previously chosen branches of the radicals (1.4) and (1.6) into account, we obtain when $t \geqslant z$

$$
\begin{aligned}
&L_t^{-1}\left\{\frac{s(e^{-R_1 z}-e^{-\omega z})}{R_1^2-\omega^2}\right\} = Q_1(\lambda,z,t) = -\frac{2}{\pi}\int_\lambda^\infty \frac{\sin(z\sqrt{y^2-\lambda^2})\sin yt}{y^2+1}dy - \\
&-\frac{2}{\pi}\int_\lambda^\infty \frac{y\sin(z\sqrt{y^2-\lambda^2})\cos yt}{y^2+1}dy + \frac{1}{\pi}\int_{\lambda^2}^\infty \frac{\sin(z\sqrt{y-\lambda^2})e^{-yt}}{y+1}dy
\end{aligned}
\tag{3.6}
$$

Using Taylor's formula, Eqs (3.752.1) and (3.742.5) from [24] and making a change of the variable of integration, we obtain

$$
\begin{aligned}
Q_1(\lambda,z,t) &= Q_1(0,z,t)+\lambda^2[I_1(\lambda_1^2)+I_2(\lambda_1^2)] = \frac{2}{\pi}\int_0^\infty \frac{\sin zx}{x^2+1}e^{-x^2 t}dx + \lambda^2\left[\frac{z}{\pi}\int_0^\infty \frac{\cos zx \sin(t\sqrt{x^2+\lambda_1^2})}{(x^2+\lambda_1^2+1)\sqrt{x^2+\lambda_1^2}}dx + \right. \\
&\left. + \frac{z}{\pi}\int_0^\infty \frac{[\cos(t\sqrt{x^2+\lambda_1^2})-e^{-(x^2+\lambda_1^2)t}]\cos zx}{x^2+\lambda_1^2+1}dx\right], \quad 0<\lambda_1<\lambda
\end{aligned}
$$

The legitimacy of differentiating the integrals in (3.6) with respect to λ^2 is easily proved. Using formula 74 (26) from [22] and the relations $z/(\sqrt{t})-2\sqrt{(t)}<0$, $(t \geqslant z)$ and $\operatorname{sh} x \leqslant xe^x$, $(x \geqslant 0)$ we will have, like (3.5)

$$
\begin{aligned}
|Q_1(0,z,t)| &= \frac{e^t}{2}\left|e^{-z}\operatorname{erfc}\left(\sqrt{t}-\frac{z}{2\sqrt{t}}\right)-e^z\operatorname{erfc}\left(\sqrt{t}+\frac{z}{2\sqrt{t}}\right)\right| = \\
&= \frac{2}{\sqrt{\pi}}e_z\int_0^\infty e^{-x^2-2x\sqrt{t}}\operatorname{sh}\frac{zx}{\sqrt{t}}dx \leqslant \frac{z}{\sqrt{\pi t}}e_z
\end{aligned}
\tag{3.7}
$$

Combining relations (3.5) and (3.6) and

$$
|I_1(\lambda_1^2)| \leqslant z\sqrt{t/2}, \quad |I_2(\lambda_1^2)| \leqslant z
$$

we obtain the fifth inequality of (3.4).

The procedure for obtaining the ninth relation of (3.4) is similar to that described in [16] for a similar original.

The last two inequalities of (3.4) are obtained by differentiation and subsequent estimation of the integral representation of $J_0(x)$ [24].

The method of obtaining estimates of δ_{ik} will be demonstrated using the example of a term from σ_{zz}. For the first term in σ_{zz} and the corresponding asymptotic representation in (2.15) we have

$$
\delta^{(1)} = L_t^{-1}\left\{\int_0^\infty \lambda f^H(\lambda)J_0(\lambda r)Q_2^*(\lambda,z,s)d\lambda\right\}
\tag{3.8}
$$

$$Q_2^*(\lambda,z,s)=\frac{R^2\chi(\lambda,\gamma)}{s^2(\omega^2-\gamma^2)}-\frac{\varepsilon^2\chi(0,\gamma)}{2(s-\gamma^2)}=Q_{21}^*+Q_{22}^*+Q_{23}^*=$$
$$=\frac{\varepsilon^2}{2(\omega^2-\gamma^2)}[\chi(\lambda,\gamma)-\chi(0,\gamma)]+\frac{\varepsilon^2}{2}\left(\frac{1}{\omega^2-\gamma^2}-\frac{1}{s-\gamma^2}\right)\chi(0,\gamma)+\frac{\lambda^2\chi(\lambda,\gamma)}{s^2(\omega^2-\gamma^2)}$$

Using the displacement theorem, the limit (3.2), the third relation of (3.4) and the convolution theorem we obtain

$$|L_t^{-1}\{Q_{21}^*\}|\le\frac{\varepsilon^2\lambda^2}{4}e^{\gamma^2 t}\left[\frac{1}{3}(t^3\eta_0+z^3\eta_1)+zt(t-z)\eta_1\right]=\lambda^2\delta_1 \tag{3.9}$$

An estimate of $L_t^{-1}\{Q_{22}^*\}$ is obtained in the same way as (3.1) and (3.9). Using (3.2) and the second inequality of (3.4) (for $\lambda=0$) we have

$$|L_t^{-1}\{Q_{22}^*\}|\le\frac{\varepsilon^2\lambda^2}{4}e^{\gamma^2 t}[t^2-(t-z)^2\eta_1+\gamma z(t-z)^2\eta_1]=\lambda^2\delta_2 \tag{3.10}$$

Using the theorem of the integration of an original, we obtain, in the same way as (3.10)

$$|L_t^{-1}\{Q_{23}^*\}|\le(\lambda^2/6)e^{\gamma^2 t}[t^3-(t-z)^3\eta_1+z(\gamma+\lambda^2 t)(t-z)^3\eta_1]=\lambda^2\delta_3+\lambda^4\delta_4 \tag{3.11}$$

From (3.8)–(3.11) we obtain the limit

$$|\delta^{(1)}|\le\beta_3(\delta_1+\delta_2+\delta_3)+\beta_5\delta_4$$

Carrying out similar calculations for the remaining terms and the remaining stresses, we finally obtain

$$\begin{aligned}
&|\delta_T|\le\beta_3(t^2/2)e^{\gamma^2 t-\gamma z}\\
&|\delta_{jj}|\le\frac{1}{2}k_j\beta_3t^2e^{\gamma^2 t-\gamma z}+\frac{1}{2}l_j\beta_3e^{\gamma^2 t}\Big\{2p_1(2)+p_3(2)+\frac{1}{3}p_3(3)+\\
&+z\left[t+\gamma(t-z)+2\kappa+\frac{8}{3\sqrt{\pi}}e_z\sqrt{t-z}\right](t-z)\eta_1\Big\}+\\
&+\frac{1}{6}w_j\beta_3e^{\gamma^2 t}\left\{2p_1(3)+\frac{16z}{5\sqrt{\pi}}e_z(t-z)^{5/2}\eta_1+z\left[\gamma+\frac{\beta_5}{\beta_3}(t+\kappa)\right](t-z)^3\eta_1\right\}+\\
&+\frac{1}{6}k_j\beta e^{\gamma^2 t}\Big\{l_j\zeta_3(t-z)^3\eta_1+\frac{\varepsilon^2}{4}\left[\zeta_3+\frac{1}{2\varepsilon}\zeta_5z(t-\varepsilon z)\right](t-\varepsilon z)^3\eta_2+\\
&+\frac{1}{20}w_j\left[\zeta_5p_2(5)+\frac{1}{2}\zeta_7zp_4(6)\right]\Big\}+\frac{1}{12}(1-k_j)\beta e^{\gamma^2 t}\Big\{\varepsilon^2\Big[\zeta_3p_2(3)+\\
&+\zeta_5z\left(z+\frac{\varepsilon^2-1}{2\varepsilon}t\right)(t-\varepsilon z)^3\eta_2\Big]+\frac{1}{10}\left[\zeta_5p_2(5)+\frac{1}{2}\zeta_7zp_4(6)\right]\Big\},\quad j=r,\varphi,z\\
&|\delta_{rz}|\le\frac{1}{2}r\beta_5e^{\gamma^2 t}\Big\{\frac{5}{6}p_2(3)+\frac{1}{16}p_2(4)+\frac{z}{3}\left(1+\frac{\beta_6}{\beta_5}\right)p_4(3)+\\
&+\frac{1}{12}\gamma\left[\frac{1}{2}p_1(4)+p_3(2)p_1(3)+\frac{1}{2}zt(2+\gamma+2\gamma t)(t-z)^3\eta_1\right]+\\
&+\frac{2}{7\sqrt{\pi}}p_1\left(\frac{7}{2}\right)+\frac{5}{16}ze_z(t-z)^3\eta_1+p_3(1)\left[\frac{8}{15\sqrt{\pi}}p_1\left(\frac{5}{2}\right)+\frac{\beta_6}{6\beta_5}p_1(3)\right]+
\end{aligned} \tag{3.12}$$

$$+\kappa z\left[\frac{8}{15\sqrt{\pi}}+\frac{\beta_6}{6\beta_5}\sqrt{t-z}\right](t-z)^{5/2}\eta_1\Bigg\}+\frac{1}{48}\beta r e^{\gamma^2 t}\left\{\zeta_5 p_2(4)+\frac{1}{2}\zeta_7 z p_4(5)\right\}$$

$$p_1(x)=t^x-(t-z)^x\eta_1,\quad p_2(x)=(t-z)^x\eta_1-(t-\varepsilon z)^x\eta_2$$

$$p_3(x)=t^x\eta_0+z^x\eta_1,\quad p_4(x)=(t-z)^x\eta_1+\frac{1}{\varepsilon}(t-\varepsilon z)^x\eta_2$$

$$w_r=w_\varphi=\frac{1}{2},\quad w_z=1,\quad \zeta_n=3\beta_n+2\beta_{n+1},\quad e_z=e^{-z^2/4t}$$

4. Let us consider an example. Suppose that in dimensionless variables $f(r)=\delta^3/(r^2+\delta^2)^{3/2}$, $\gamma=2.5\times10^{-9}$, $h=1.4\times10^{-6}$, $\nu=0.3$, and $\delta=4\times10^9$. Then $f^H(\lambda)=\delta^2e^{-\lambda\delta}$, $A_{mn}(r)$ can be expressed in terms of elementary functions [24], $\beta_k=k!/\delta^{k-1}$, $k=1,2,\ldots$. For $t=t_0=1.5\times10^8$ and $r=\delta/2$, $0\leqslant z\leqslant 2t_0$ the relative errors of the asymptotic representations of the normal stresses (2.15), calculated by (3.12), do not exceed 5%.

For σ_{rz} from (2.15), Eq. (3.12) gives a satisfactory relative error (of up to 8%) in the neighbourhoods of the elastic wavefronts.

Calculations show that for all $0<t\leqslant t_0$ the maximum values of the normal stresses are three orders of magnitude larger than the maximum values of σ_{rz}. consequently, in calculations connected with an investigation of the stress state at short heating times, we can use the solution of the corresponding one-dimensional problem multiplied by $f(r)$.

5. Part of the originals from (2.15) is available in tables [19, 22]. The remaining originals are given below

$$L_t^{-1}\left\{\frac{e^{-z\sqrt{s}}}{s(s-\gamma^2)(\sqrt{s}+h)}\right\}=\frac{1}{\gamma^2}\left\{\frac{1}{2(\gamma+h)}e^{\gamma^2t-\gamma z}\operatorname{erfc}\left(\frac{z}{2\sqrt{t}}-\gamma\sqrt{t}\right)-\right.$$

$$\left.-\frac{1}{2(\gamma-h)}e^{\gamma^2t+\gamma z}\operatorname{erfc}\left(\frac{z}{2\sqrt{t}}+\gamma\sqrt{t}\right)-\frac{1}{h}\operatorname{erfc}\frac{z}{2\sqrt{t}}\right\}+\frac{1}{h(\gamma^2-h^2)}e^{h^2t+hz}\operatorname{erfc}\left(\frac{z}{2\sqrt{t}}+h\sqrt{t}\right).$$

$$L_t^{-1}\left\{\frac{e^{-z\sqrt{s}}}{(s-\gamma^2)(s-1)(\sqrt{s}+h)}\right\}=\frac{1}{\gamma^2-1}\left\{\frac{1}{2(\gamma+h)}e^{\gamma^2t-\gamma z}\operatorname{erfc}\left(\frac{z}{2\sqrt{t}}-\gamma\sqrt{t}\right)-\right.$$

$$-\frac{1}{2(\gamma-h)}e^{\gamma^2t+\gamma z}\operatorname{erfc}\left(\frac{z}{2\sqrt{t}}+\gamma\sqrt{t}\right)-\frac{1}{2(h+1)}e^{t-z}\operatorname{erfc}\left(\frac{z}{2\sqrt{t}}-\sqrt{t}\right)-$$

$$\left.-\frac{1}{2(h-1)}e^{t+z}\operatorname{erfc}\left(\frac{z}{2\sqrt{t}}+\sqrt{t}\right)\right\}+\frac{h}{(h^2-1)(\gamma^2-h^2)}e^{h^2t+hz}\operatorname{erfc}\left(\frac{z}{2\sqrt{t}}+h\sqrt{t}\right).$$

$$L_t^{-1}\left\{\frac{e^{-z\sqrt{s}}}{s\sqrt{s}(s-\gamma^2)(s-1)(\sqrt{s}+h)}\right\}=\frac{2}{\gamma^2h}\sqrt{\frac{t}{\pi}}e^{-z^2/(4t)}-\frac{1}{\gamma^2h}\left(z+\frac{1}{h}\right)\operatorname{erfc}\frac{z}{2\sqrt{t}}-$$

$$-\frac{1}{h^2(h^2-1)(\gamma^2-h^2)}e^{h^2t+hz}\operatorname{erfc}\left(\frac{z}{2\sqrt{t}}+h\sqrt{t}\right)-\frac{1}{2(\gamma^2-1)}\left\{\frac{1}{h+1}e^{t-z}\operatorname{erfc}\left(\frac{z}{2\sqrt{t}}-\sqrt{t}\right)-\right.$$

$$-\frac{1}{h-1}e^{t+z}\operatorname{erfc}\left(\frac{z}{2\sqrt{t}}+\sqrt{t}\right)-\frac{1}{\gamma^3(\gamma+h)}e^{\gamma^2t-\gamma z}\operatorname{erfc}\left(\frac{z}{2\sqrt{t}}-\gamma\sqrt{t}\right)-$$

$$\left.-\frac{1}{\gamma^3(\gamma-h)}e^{\gamma^2t+\gamma z}\operatorname{erfc}\left(\frac{z}{2\sqrt{t}}+\gamma\sqrt{t}\right)\right\}$$

$$L_t^{-1}\left\{\frac{1}{s\sqrt{s}(s-\gamma^2)(\sqrt{s}+1)(\sqrt{s}+h)}\right\}=-\frac{2}{\gamma^2h}\sqrt{\frac{t}{\pi}}+\frac{h+1}{\gamma^2h^2}+\frac{1}{\gamma^3(\gamma+1)(\gamma+h)}e^{\gamma^2t}-$$

$$-\frac{\gamma^2+h}{\gamma^3(\gamma^2-1)(\gamma^2-h^2)}e^{\gamma^2t}\operatorname{erfc}\gamma\sqrt{t}-\frac{1}{(\gamma^2-1)(h-1)}e^t\operatorname{erfc}\sqrt{t}+\frac{1}{h^2(\gamma^2-h^2)(h-1)}e^{h^2t}\operatorname{erfc}h\sqrt{t}$$

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